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EXACT TRAVELING WAVE SOLUTIONS FOR THE GENERALIZED HIROTA-SATSUMA COUPLE KDV SYSTEM USING THE EXP$\left(-\phi(\zeta)\right)$ -EXPANSION METHOD

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ABSTRACT

In this research, we find the exact traveling wave solutions involving parameters of the generalized Hirota-Satsuma couple KdV system according to the $\boldsymbol{exp(-\varphi(\xi))}$ -expansion method and when these parameters are taken to be special values we can obtain the solitary wave solutions which is derived from the exact traveling wave solutions. It is shown that the proposed method provides a more powerful mathematical tool for constructing exact traveling wave solutions for many other nonlinear evolution equations.
Keywords: The $\boldsymbol{exp(-\varphi(\xi))}$ -expansion method; The generalized Hirota-Satsuma couple KdV system; Traveling wave solutions; Solitary wave solutions.

INTRODUCTION

No one can deny the important role which played by the nonlinear partial differential equations in the description of many and a wide variety of phenomena not only in physical phenomena, but also in plasma, fluid mechanics, optical fibers, solid state physics, chemical kinetics and geochemistry phenomena. So that, during the past five decades, a lot of method was discovered by a diverse group of scientists to solve the nonlinear partial differential equations. Such methods are tanh - sech method [1]-[3], extended tanh - method [4]-[6], sine - cosine method [7]-[9], homogeneous balance method [10, 11],F-expansion method [12]-[14], exp-function method [15, 16], trigonometric function series method [17], (G'/G)-expansion method [18]-[21], Jacobi elliptic function method [22]-[25], The $\boldsymbol{exp(-\varphi(\xi))}$ -expansion method [26]-[28] and so on.

The objective of this article is to apply the $\boldsymbol{exp(-\varphi(\xi))}$ -expansion method for finding the exact traveling wave solution of the generalized Hirota-Satsuma couple KdV system which plays an important role in mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of the $\boldsymbol{exp(-\varphi(\xi))}$ -expansion method In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 5, conclusions are given.

DESCRIPTION OF METHOD

Let us we have the following nonlinear evolution equation

$$P(u, u_t, u_x, u_{tt}, u_{xx}, \ldots\ldots) = 0, \quad (2.1)$$

Since, P is a polynomial in $u(x,t)$ and its partial derivatives. In the following, we give the main steps of this method

Step 1. We use the traveling wave solution in the form

$$u(x,t) = u(\xi), \qquad \xi = x - ct, \qquad (2.2)$$

Where c is a positive constant, to reduce Eq. (2.1) to the following ODE:

$$p(u, u', u'', u''', \ldots\ldots) = 0, \qquad (2.3)$$

Where P is a polynomial in u (ξ) and its total derivatives.

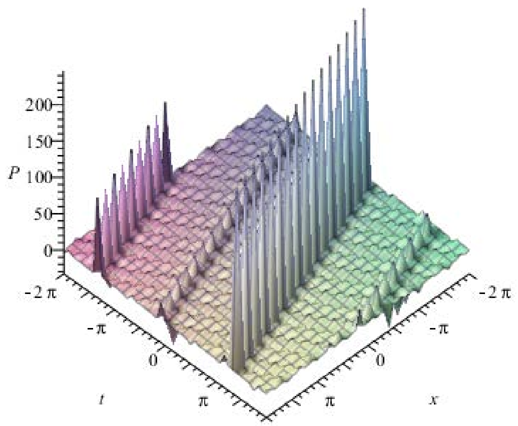
Step 2. Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $exp(-\varphi(\xi))$ as follow

$$u(\xi) = a_m(\exp(-\varphi(\xi))^m + \cdots, a_m \neq 0 \quad (2.4)$$

Where $\varphi(\xi)$ satisfies the ODE in the form

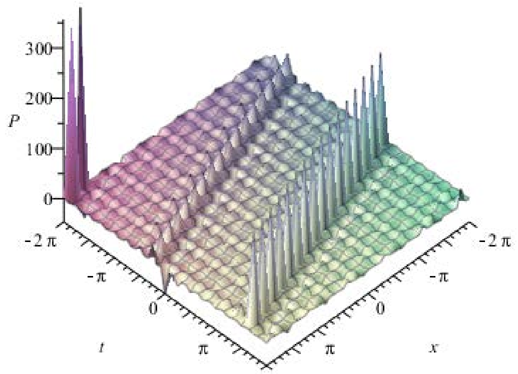
$$\varphi'(\xi) = \exp\left(-\varphi(\xi)\right) + \mu \exp\left(\varphi(\xi)\right) + \lambda, \quad (2.5) \qquad (2.5)$$

The solutions of ODE (2.5) are



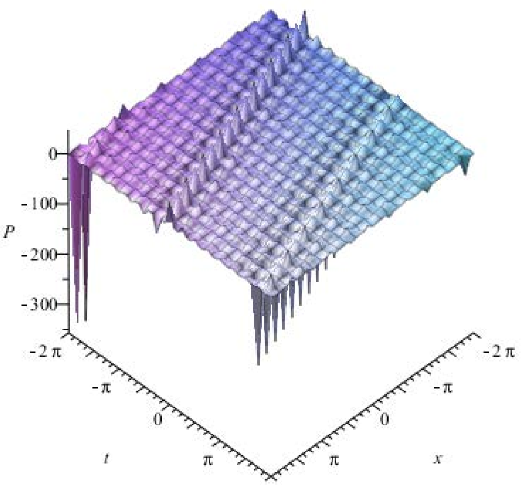
P_{14}, *Case* 1.

Eq. (3.18)



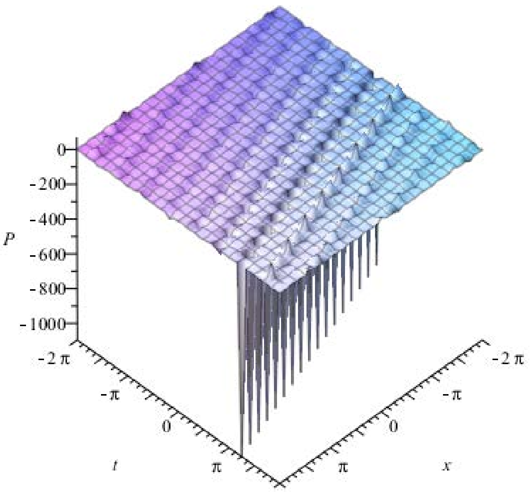
P_{13}, *Case* 2.

Eq. (3.18)



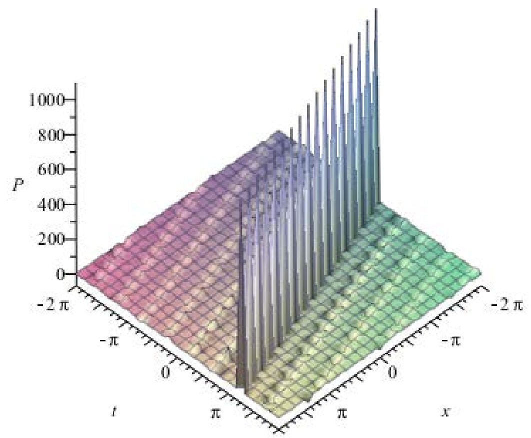
P_{14}, *Case* 2.

Eq. (3.18)



P_{13}, *Case* 3.

Eq. (3.18)



P_{14}, *Case* 3.

Eq. (3.18)

5 CONCLUSIONS

The $\boldsymbol{exp}(-\boldsymbol{\varphi}(\boldsymbol{\xi}))$ -expansion method has been applied in this paper to find the exact traveling wave solutions and then the solitary wave solutions of the generalized Hirota-Satsuma couple KdV system. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of nonlinear dynamics of the generalized Hirota-Satsuma couple KdV system are new and different from those obtained in [32], and figs. [1-16], show the solitary traveling wave solution of the generalized Hirota-Satsuma couple KdV system . We can conclude that the $\boldsymbol{exp}(-\boldsymbol{\varphi}(\boldsymbol{\xi}))$ -expansion method is

is a very powerful and efficient technique in finding exact solutions for wide classes of nonlinear problems and can be applied to many other nonlinear evolution equations in mathematical physics. Another possible merit is that the reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

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